THE PHYSICAL BASIS OF Lg GENERATION BY EXPLOSION SOURCES

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Sponsored by National Nuclear Security Administration
Office of Nonproliferation Research and Engineering
Office of Defense Nuclear Nonproliferation

Contract No. DE-FC03-02SF22676

ABSTRACT

The goal of the project is to develop a quantitative predictive capability for explosion-generated Lg phases with a sound and unambiguous physical basis. The research program consists of a theoretical investigation of explosion-generated Lg combined with an observational study. The specific question addressed by this research program is how the Lg phase is generated by underground nuclear explosions. This question is fundamental to how Lg phases are interpreted for use in explosion yield estimation and earthquake/explosion discrimination.

We have been looking initially at several explosion data sets - 1) Degelen Mountain explosions recorded at distances less than 100 km, and corresponding recordings at Borovoye at a distance of approximately 650 km; 2) recordings from Russian deep seismic sounding experiments; 3) Nevada Test Site (NTS) explosion sources including the Nonproliferation Experiment (NPE) and nuclear tests covering a range of source depths and media properties. A particularly interesting result from the Degelen explosion data is that:

a. The near regional data has small Sg phases, distinct from the strong Rg phase that persists out to at least 50 km.
b. At Borovoye, there are strong Sn and Lg phases.

Modeling of these data shows that both results are consistent with the signals from a shallow, axisymmetric compensated linear vector dipole (CLVD) source. This source has an S node in the horizontal direction, and therefore makes only a small direct S wave at the near regional stations, however it is a strong generator of S at takeoff angles corresponding to the crustal phases Lg and Sn. Synthetic seismograms generated using wavenumber integration are a very good match to the observed local Sg and regional Sn and Lg phases.

We are considering four candidate mechanisms for explosion-generated Lg:

1. Direct generation by the explosion source, where the explosion is modeled as a point compressional source.
2. Secondary generation by the explosion source, where Lg is generated primarily by the nonspherical parts of the explosion source, with strong influence from the free surface.
3. Rg scattering. The hypothesis is that the Rg phase is scattered as it travels away from the explosion and is converted to Lg.
4. P scattering. The hypothesis is that Lg is in a sense a variant on P coda containing converted P->S phases in the crust.

The observations discussed above favor explanation number 2. The spherically symmetric part of an explosion source in a high velocity medium generates very little Lg, and therefore does not explain the observations. However, the observed Lg waveforms and amplitudes can be explained by adding a CLVD, which is the lowest order non-spherical correction to the spherical source. That Rg persists to such large distances argues against explanation 3. We are in the process of performing a series of 2D nonlinear calculations of explosion sources. The main purpose of this work is to quantify the amount of seismic radiation generated by the non-spherical parts of a realistic explosion source. We have completed three calculations – a 2D nonlinear calculation modeled after the NPE source and structure, and two calculations of Degelen Mountain explosions. Lg calculated from the NPE calculation is consistent with an explosion plus a CLVD source with about half the strength of the explosion. However, because of the very low velocities at the NPE source location, the explosion generates substantial Lg directly and this alone may be sufficient to explain the Lg observations in this case. Since Degelen consists of a high velocity granite, little Lg is generated directly by the explosion, so the CLVD component is much more important.

OBJECTIVE

The objective of this project is to develop a quantitative predictive capability for explosion-generated Lg phases with a sound physical basis. The research program consists of a theoretical investigation of explosion-generated Lg combined with an observational study. The specific question addressed by this research program is how the Lg phase is generated by underground nuclear explosions. This question is fundamental to how Lg phases are interpreted for use in explosion yield estimation and earthquake/explosion discrimination.

RESEARCH ACCOMPLISHED

The importance of Lg to nuclear monitoring arises from Nuttli's (1986) remarkable results about the consistency of the Lg/yield relationship for explosions and from the application of the P/Lg discriminant (e.g. Bennett et al, 1997a, 1997b) and a variety of spectral ratio discriminants. Although Lg based discriminants have been widely used for regional earthquake/explosion discrimination, they are not completely satisfactory because regional variations are large enough that the discriminants cannot be regarded as reliable in uncalibrated regions. Because of this difficulty with transportability, it is very important to understand how Lg is generated in order to identify conditions under which the discriminants will fail and how to correct for those failures.

One of the peculiar properties of Lg is that it does not appear to be generated primarily by the explosion, even though it correlates very well with yield. That is, calculations based on point, spherical explosions in plane-layered media, which are generally satisfactory with minor corrections for predictions of other explosion-generated seismic phases, predict Lg that is both much smaller and more dependent on earth structure than the observations. Instead, Lg appears to be generated primarily by the non-spherical components of the explosion source including spall and other free surface effects. The most widely used explanation for Lg suggests that these non-spherical components can be modeled as a compensated linear vector dipole (CLVD) source, which generates Rg (regional fundamental mode surface waves), which then scatters into Lg (e.g. Gupta et al, 1997; Patton and Taylor, 1995; Patton, 2001). However, the non-spherical components of the source also generate Lg directly, so it may not be necessary to go through the intermediate Rg phase.

In this study, we are using numerical techniques coupled with observations of Lg and a review of observations from the literature to try to establish the physical mechanism for explosion-generated Lg. That is, we are looking at Lg data from explosions, performing numerical calculations of explosions, and comparing predictions with observations. Significant accomplishments to date include:

1. A detailed review of the literature on Lg generation, scattering, and related topics.
2. Nonlinear finite difference calculations of three explosions – the NPE and two Degelen Mt. explosions.
3. Collection and review of explosion-generated data from a variety of sources, including NTS data from LLNL stations, lines of stations that recorded Soviet deep seismic sounding (DSS) data, near field and near regional recordings of Degelen mountain explosions, and data from the same events recorded at Borovoye.

In this paper, we start by reviewing nonlinear modeling of explosion sources, and the calculations performed in the current study. Then we discuss the data that we are using for analysis, and finally compare results from the calculations with some of the observations.

The Nonlinear Explosion Source Near the Free Surface

The first approximation for an explosion is usually taken to be a point, spherically symmetric dilatational source. While this would be correct for an explosion in an infinite, uniform medium, the presence of the earth surface renders that description inadequate. The explosion source can be modeled more realistically using nonlinear finite difference calculations of explosions in a realistic earth model with gravity (Stevens et al., 1991). This approach models all near source effects including spall, cracking, and nonlinear deformation.

The seismic source generated by a realistic explosion can be described in various ways. Since a large part of the deformation is related to spallation of the surface layers, a natural characterization of the source is in terms of a horizontal tension crack at a shallower depth than the compressional source (Day and McLaughlin, 1991). The source can also be described by a multipolar expansion (e.g. Stevens, 1980), which for axisymmetric configurations includes a spherically symmetric term, a CLVD term, and higher order terms. Since the spherically symmetric term generates little Lg, it is reasonable to expect that the CLVD term would characterize the complex explosion source for Lg. Patton and Taylor (1995) suggest that this is the case and that the CLVD source is more consistent with the data than the tensile crack source. Gupta and Wagner (1998) point to spectral nulls in Lg from NTS explosions as evidence that Lg is generated by Rg scattering from the CLVD source. It is puzzling, however, why the nulls are not filled in by Rg from the direct explosion, which generates a strong Rg phase as long as it is less than a wavelength deep.

In this study we have performed new nonlinear calculations in two very different types of structures: first, the Non-Proliferation Experiment (NPE), which was conducted at the Nevada Test Site (NTS) in a very low velocity medium; and second, a typical Degelen Mountain explosion which took place in a high velocity granite medium. Descriptions of these calculations follow.

NPE calculation

We performed a calculation of the Non-Proliferation Experiment (NPE) using detailed rock properties from Rimer et al. (1994). The NPE was a chemical explosion with yield equivalent to one kiloton of TNT. The material geology at the NPE site is based on that of the nearby Misty Echo event and consists of 4 layers (Figure 1a). Layers 1 and 4 are nonporous, and layers 2 and 3 have porosities of 3% and 0.5% respectively. The explosive is in a cylindrical cavity centered at 389 m depth, and is 7.7 m in radius (horizontally) and 5.2 m in height (vertically). We use a two-dimensional axisymmetric Lagrangian finite difference code to simulate nonlinear wave propagation from tamped underground explosions. The code has been used successfully for many ground motion calculations in our previous work. In Figure 1 (left) we show the extent of yielding and cracking from an HE source of 1.315kt of a 50/50 ANFO/emulsion mix. The region of nonlinear deformation near the explosion is approximately spherical, but elongated and slightly offset vertically. The region of cracking is confined to near the free surface. This explosion is deeply overburied, so there is less asymmetry than in a normally buried explosion.

We calculated full waveform synthetics from this calculation at a distance of 400 km. The synthetics are derived by saving the stresses and displacements on a monitoring surface outside the nonlinear region near the explosion and integrating with a Green's function calculated using wavenumber integration. Figure 1 (right) shows the synthetics together with synthetics from a point explosion and a CLVD source. Because this is a very low velocity structure, the crust effectively traps P->S converted waves from the explosion, so in this case the full waveform and in particular the Lg phase from the complex explosion source is modeled quite well by a point explosion source.

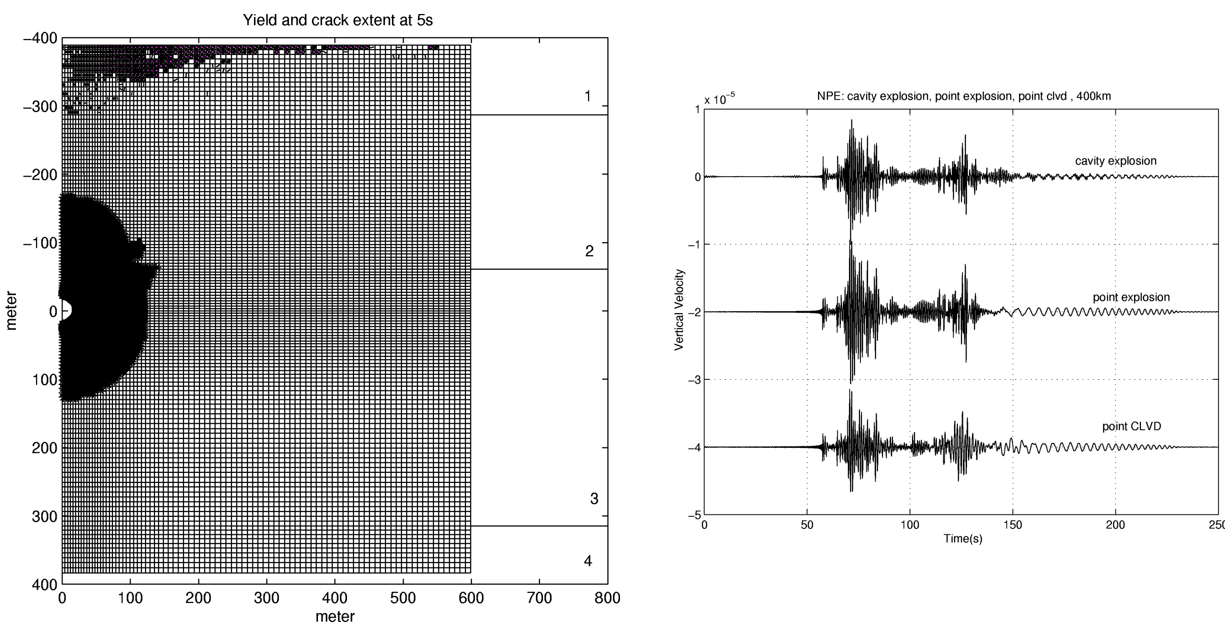

Figure 1. Region of nonlinear deformation and cracking from the NPE calculation (left). Vertical component synthetic seismograms at 400km from the HE explosion (right, top), and from a point explosion (right, middle) and point CLVD (right, bottom) in the same structure and at the same depth.

Degelen Mountain calculation

We performed two calculations for nuclear explosions at the Degelen Mountain test site. Material properties were taken from a detailed study of Degelen explosions (Stevens et al., 2003), and calculations were performed for the "wide pulse" and "narrow pulse" models described in that study. Figure 2 shows the results for the wide pulse model. The Degelen site consists of granite with a P-wave speed of 5.1km/s in the top 1.3km. In this calculation, the initial cavity is spherical, 7.3m in diameter, and its center is 300m deep. The "cavity" in this case corresponds to the region vaporized by the 62 kt nuclear explosion. The shape of the region of elastic deformation (Figure 2, left) is much different from the NPE and more typical of explosions at or above standard containment depth (Standard containment depth is approximately $122Y^{1/3}$ meters where Y is yield in kilotons (Murphy 1977), or 480 meters in this case). Note that the nonlinear region is more conical than spherical.

As before, full waveform synthetic seismograms were computed for this calculation and compared with synthetics from a point explosion source and a point CLVD source. In this case, synthetics were calculated at 650 km, the distance from Degelen to Borovoye. Figure 2 (right) shows the synthetics from the three sources. Because the Degelen test site is a high velocity structure, P->S converted waves are not trapped in the crust, so the point explosion generates only a small Lg phase, however the asymmetric part of the source which can be modeled approximately as a CLVD source generates shear waves directly, and therefore generates stronger Lg as well as a sharp Sn phase. It is clear from Figure 2 (right) that the CLVD source dominates the regional seismogram, and more detailed analysis shows that the complete source can be modeled quite well as a point explosion plus a CLVD with half the explosion moment.

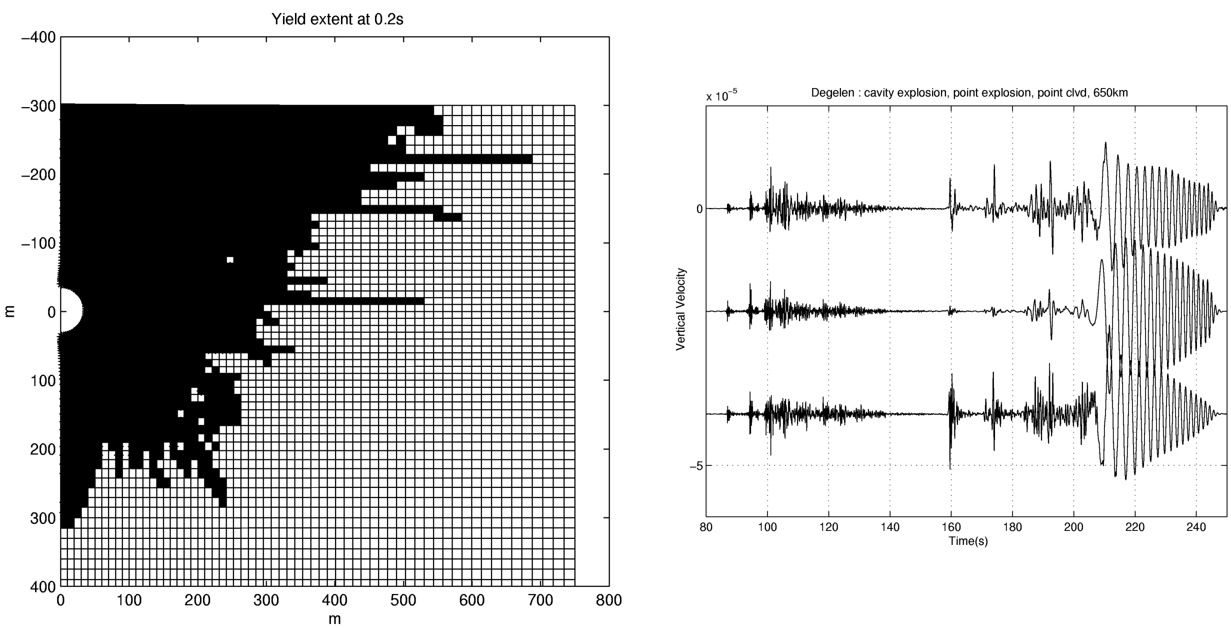

Figure 2. Region of nonlinear deformation from the Degelen Mountain calculation (left). Vertical component synthetic seismograms at 650km from the nuclear explosion (right, top), and from a point explosion (right, middle) and point CLVD in the same structure and at the same depth.

Data

We have collected several data sets that permit observation of the evolution of explosion shear wave phases from very near the source to regional distances.

1) Degelen-Borovoye: This dataset includes 19 radial and 14 vertical recordings at 7-57 km distance from 7 explosions at the Degelen test site, for which good 3-component records are also available from Borovoye (BOR), 650 km from Degelen.

Table 1. The number of local radial and vertical recordings extending past the predicted Sg time for Degelen events, for which there are also Borovoye seismic records. Where there are two depths of burial shown, the smaller indicates slant range to a free surface.

Event			Local Records	
Date	Yield (Kt)	Depth of burial, m	No. Extending to Predicted Sg Time	Distance range, km
1971/12/15	1.5	115/145	5 radial, 4 vertical	7-33
1987/07/17	78	267	4 radial, 4 vertical	14-50
1987/12/20	3.2	105	2 radial	13-17
1988/04/22	2.3	124	1 radial	57
1988/10/18	2.5	125	2 radial, 1 vertical	11-17
1988/11/23	19	204	3 radial, 2 vertical	14-22
1989/10/04	1.8	94	4 radial, 3 vertical	16-42

2) Deep Seismic Sounding (DSS) PNEs: DSS records from IRIS (Morozov et al., 2001) for 6 PNEs in the former Soviet Union provide 3 component recordings every 10 to 15 km, from 100s of meters to 1000s of km from nuclear explosions in different media. We have performed extensive quality control and preliminary analyses on 4 of these data sets (Table 2).

Table 2. Quartz and Craton PNEs for which we have performed QC and preliminary analyses

Date	Time	Lat	Lon	Depth	Yield	Name
1984-Aug-11	19:00:00.20	65.05	55.10	759 m	8.5 Kt	Quartz-2
1984-Aug-25	19:00:00.33	61.90	72.10	726 m	8.5 Kt	Quartz-3
1984-Sept-17	21:00:00.0	55.87	87.446	557 m	10 Kt	Quartz-4
1978-Aug-24	18:00:00.0	66.60	86.21	-	22 Kt	Craton-2

3) Lop Nor: For the Chinese test site at Lop Nor, we have collected digital waveforms at regional stations 250 to 1900 km from 10 nuclear explosions, which range from m_b 4.7 to 6.4. We are reviewing these data to determine their quality and value for use in this project. Records from the same stations are available for earthquakes in the vicinity of Lop Nor for use in comparing excitation differences between source types.

4) NPE: This data set includes 3-component records at 4, 39, and 55 km, thirty-three 3-component regional seismograms from 200 to 1000 km and a range of azimuths (from IRIS, LLNL, and SNL), plus 3-component records from a line of regularly spaced instruments extending from 80 to 274 km eastward of the source (from Univ. of Az.).

5) NTS: We have high quality, 3-component records from LLNL stations at 190 to 400 km for 155 NTS nuclear explosions (Table 3). The explosions span the Yucca, Pahute, and Rainier test areas and a wide range in yield, mb, depth of burial, and porosity, enabling analyses of Lg generation sensitivity to source parameters. The Lg signals suggest differences between sources. To characterize the explosion Lg signals at these stations, we have been performing a variety of parametric measurements, including amplitude, spectral and filter processing.

Table 3. Summary of regional NTS events for which LLNL station data are being analyzed

NTS Source Areas	Number of Events	mb	Depths (m)	Distance range (km)
Yucca	100	3.7-5.8	200-701	220-400
Pahute	40	4.8-5.7	111-680	190-400
Rainier	15	4.0-5.0	261-400	200-400

Observations

There is a set of observations that hold for most of the Degelen and DSS explosion records analyzed thus far.

1) There are small but distinct Sg arrivals at less than 10 km to 10s of km from explosions. The Sg amplitude appears to increase with distance from the source.
2) At regional distances, there are large impulsive Sn arrivals.
3) At regional distances, there are large Lg arrivals.
4) At all distances, the tangential component S-wave phases are as large as those on the radial components.
5) The horizontal component Sn and Lg are much larger than their vertical component counterparts.
6) There is large high frequency Rg out to 100s of km distance.

Local radial and vertical records for a 1.5 Kt Degelen explosion, with the Rg phase included (left column) and with Rg cut off (right) show a small but distinct arrival on each record, marked by an S-wave pick (labeled ISUO) (Figure 3). The S minus P times are all consistent with the known velocity structure at Degelen, indicating that the Sg comes either directly from the source or is scattered almost instantly. The S-wave is minuscule at 7 km distance, and grows relative to P with distance. It is clearly a separate shear wave phase from the very large Rg phase. Recordings 645 km from the same explosion, at BOR, show impulsive Sn and large Lg on all three components (Figure 4).

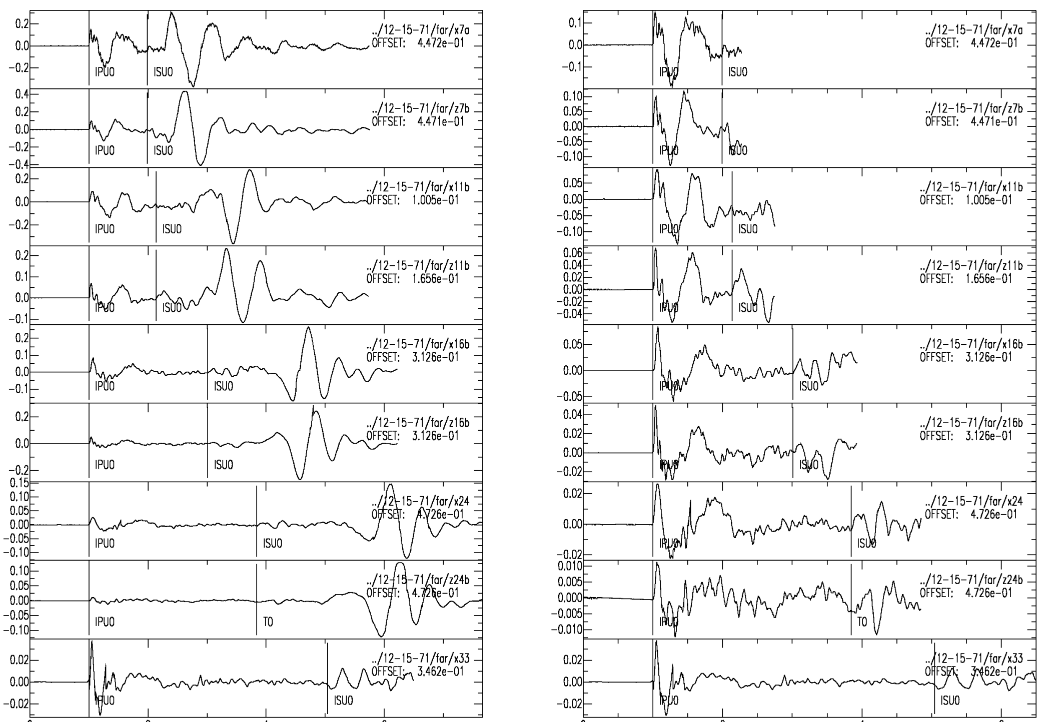

Figure 3: Local radial and vertical records for the 1.5 Kt, 12/15/71 Degelen explosion, aligned on the P-wave arrival, with the S-wave arrival marked (labeled ISUO). Rg is included on the left and excluded on the right. Distances and components are in the filenames, to the upper right of each trace, e.g. x is radial, z is vertical, the number is distance in km.

Seismograms recorded 597 km from Quartz-4 have much larger horizontal component shear wave amplitudes than their vertical counterparts (Figure 5). Sn and Lg are not visible in the vertical record filtered from 1 to 10 Hz (bottom trace of upper plot), but are apparent in the horizontals. Sn, arriving at ~4.1 km/s is visible only in the filtered data. Lg is most prominent in the records filtered from 0.5 to 1 Hz (lower 3 traces). This typical of DSS PNE records at all distances, as record sections (Figure 6) and spectrograms (Figure 7) of near regional records show.

The largest arrivals at the predicted Sg/Lg times on the spectrograms of records from 138 and 168 km east of Quartz-3 (Figure 7) are on the tangential components. The begin and end time of these phases on the tangential components yield group velocities of 3.35 to 2.5 km/sec at 138 km and 3.3 to 2.9 at 168 km, consistent with earlier observations of late arriving Lg from shallow events. The radials also have large arrivals at the same time, but the phase is completely missing on the verticals. There are small arrivals on the verticals, at ~2.3 to 2.4 km/s, before the Rg but after the horizontal component arrivals. As with Quartz-4 (Figure 5), the shear waves, here Sg or Lg, are very large on the horizontal components and practically nonexistent on the verticals.

Analysis

The existence of a distinct pickable arrival at the predicted Sg time and its separation from Rg for the full range of local distances are inconsistent with its being scattered from Rg. The persistence of large Rg to 100s of kms (Figures 3 and 7) and the impulsiveness of Sn (Figure 4) also argue against gradual scattering of Rg to other S wave phases.

The lack of high frequency shear waves on vertical but not horizontal DSS records, and the substantial difference at lower frequency, casts doubt on much of what has been thought to be known about Lg, as nearly all previous work on Lg used only vertical component recordings.

The velocity structure at the Degelen test site is high velocity all the way to the surface. Using that structure, a CLVD source at 120 m depth reproduces the small, local Sg phases much more effectively than a pure explosion source of twice the CLVD source size (Figure 8). The synthetics are convolved with a 0.125 second wide source function (triangle).

The 120 m source depth is appropriate for most of the Degelen explosion source depths. Regional synthetics are surprisingly similar for the two source types at 120 m. The explosion and CLVD synthetics have comparable Lg amplitudes, and the explosion even produces a small Sn, although it is much smaller than that of the CLVD. Explosion amplitude is scaled to match observed initial P amplitude, and the CLVD is half the explosion size. Figure 9 shows vertical synthetic seismograms for the same model and sources at 120, 300, and 800 m depth, compared with the BOR vertical seismogram from the same 12-15-71 explosion.

The middle and right columns of Figure 9 show the effects of depth on the synthetics. The 300 m depth sources are slightly deeper than the largest Degelen explosion. The 300 m CLVD source produces slightly larger Lg and much larger Sn than the explosion source. The 800 m source depth is similar to the deeper DSS sources. At that source depth, the explosion Sn is minuscule and the Lg quite small, while those phases dominate the CLVD synthetics.

Another important factor not addressed here is the effect of the high velocity source structure of Degelen used in the 1-D modeling. The propagation path more typically has lower near surface velocities, which increase the Lg amplitude of the synthetics. That effect will be addressed in future work.

CONCLUSIONS AND RECOMMENDATIONS

The results to date suggest that Lg and Sn phases are generated by the explosion and interaction with the near surface media, without a significant contribution from secondary sources such as Rg scattering, and that these phases can be predicted fairly well with a realistic model of the explosion source that includes nonlinear effects and the free surface interaction. The observed persistence of Rg to large distances also argues against Rg scattering as a significant contributor to Lg. If the source-generated Lg hypothesis is correct, it allows some predictions to be made that will be considered further in the continuing project through comparison with observations. For example, the dependence of Lg on source media, source region structure, and depth should be predictable. As can be seen from the calculations for the NPE and Degelen, even explosions in dramatically different media and source regions generate significant Lg, but may have significantly different composition due to differences in modal structure and the relative excitation caused by explosion and CLVD components.

More difficult to explain is the observed presence of Lg on the tangential components. This cannot be modeled with an axisymmetric source, and must be due to either a non-axisymmetric source effect, or to polarization changes along the source to receiver path. If the former, then the tangential Lg should have an observable radiation pattern and is unlikely to correlate as well with yield as the vertical component. That tangential Sg is observed at quite close range favors the first explanation.

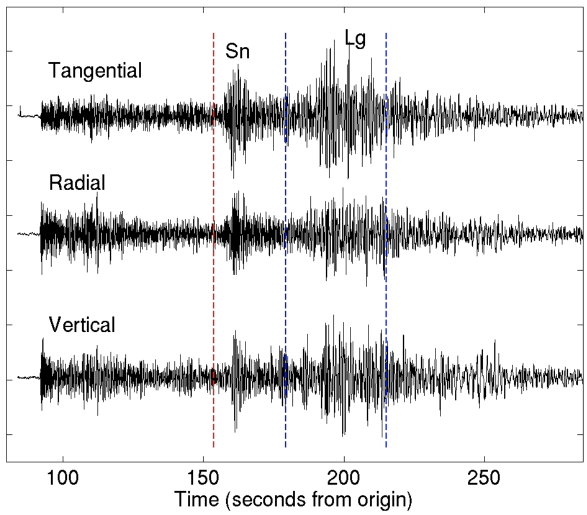

Figure 4: Tangential, radial, and vertical (from top down) records at BOR, 645 km from Degelen. The impulsive Sn arrives at ~4.1 km/s (1st line is at 4.2 km/s). The 2nd and 3rd lines, at 3.6 and 3.0 km/s, bracket the large Lg.

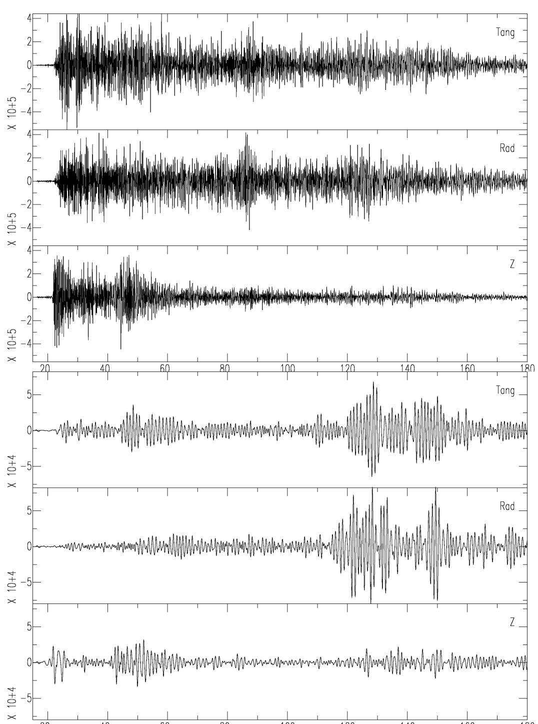

Figure 5: Tangential, radial, and vertical (from top down) records 597 km west of Quartz-4. The upper 3 traces are filtered from 1-10 Hz, and the lower 3 are filtered from 0.5-1 Hz. Time axis is reduced by range/10.

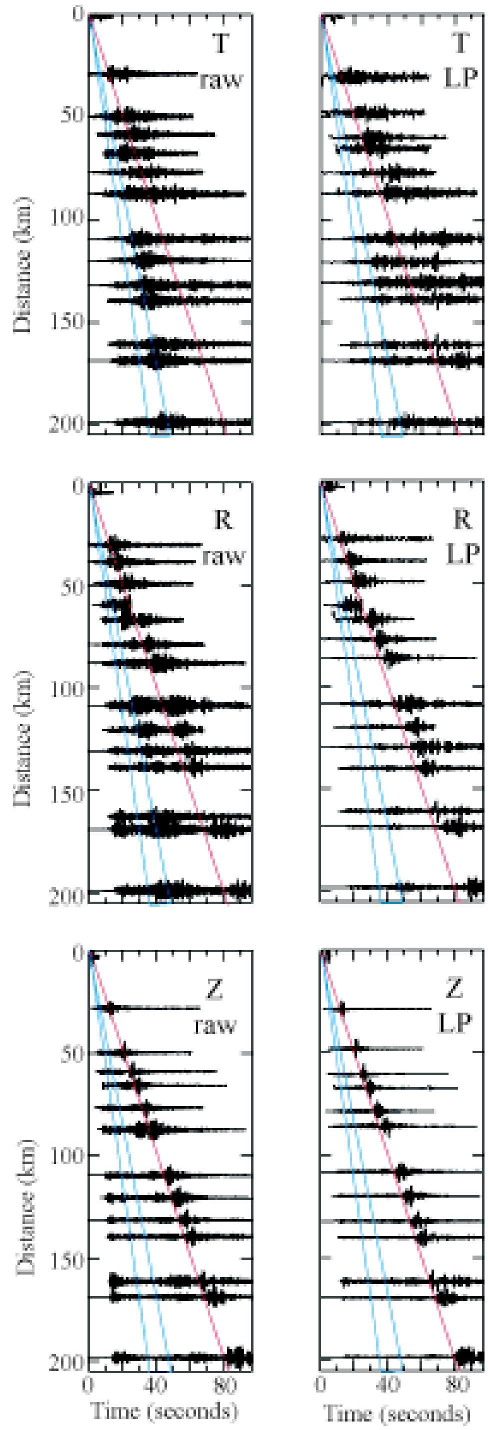

Figure 6: Tangential, radial, and vertical (from top down) record sections to 200 km east of Quartz-3. Raw traces are on the left and filtered (0.1-1.0 Hz) on the right. Amplitudes are scaled by the maximum of each trace. Blue lines are 3.6 to 3.0 km/s group velocity and red is 2.0 km/s. The time axis is reduced by range/10.

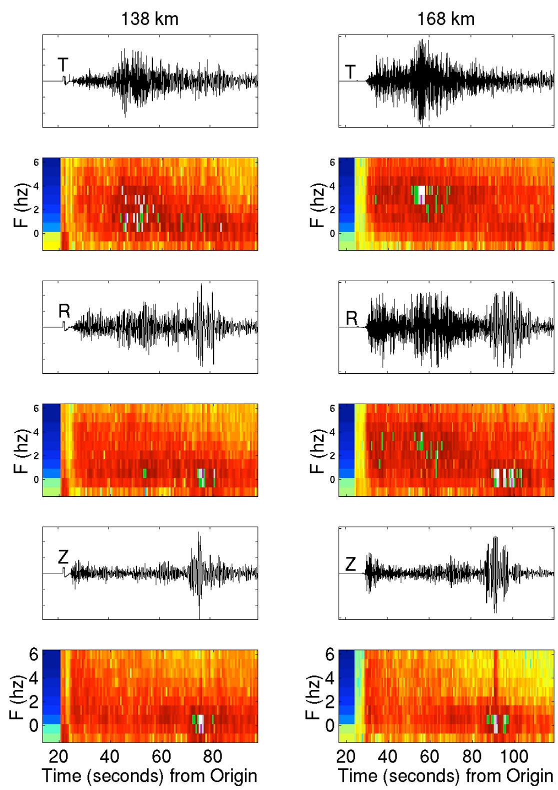

Figure 7: Spectrograms of the tangential, radial, and vertical records (from top down) 138 (left) and 168 (right) km east of Quartz 3.

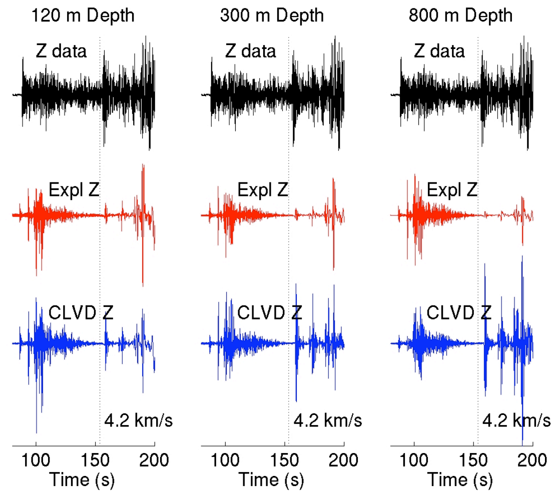

Figure 9. Vertical seismograms recorded at BOR for the 12-15-71 explosion (top), and explosion (middle) and CLVD (bottom) synthetics for source depths of 120 (left), 300 (middle), and 800 (right) m.

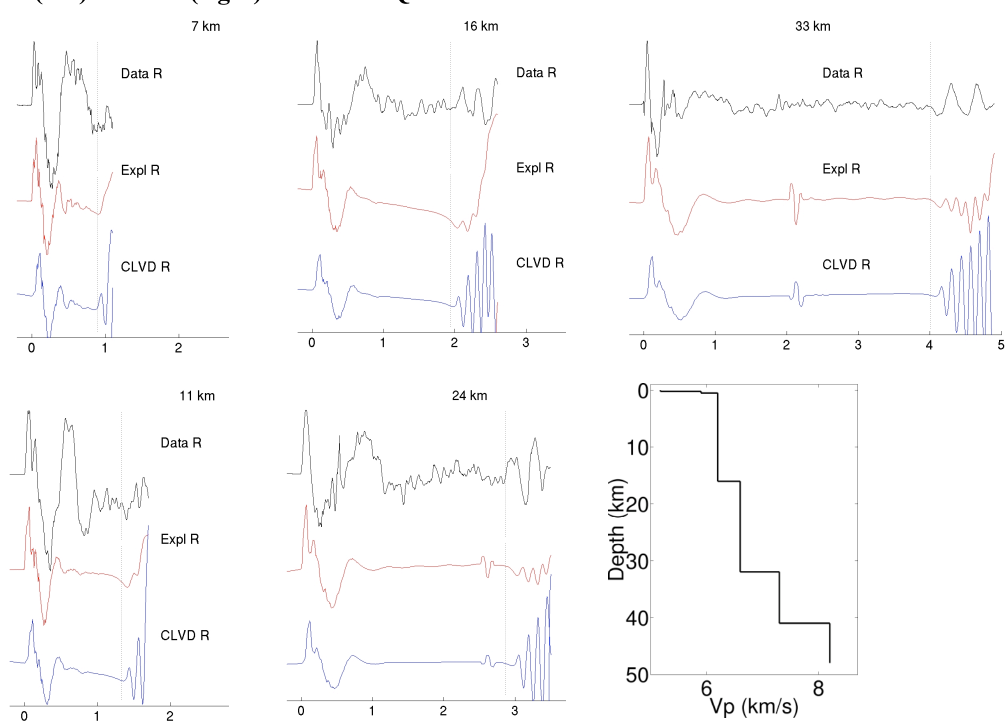

Figure 8: Radial data (top), explosion synthetics (middle), and CLVD synthetics (bottom) for the 12-15-71 Degelen explosion. Velocity structure used is shown in the lower right.

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