

**SEISMIC EVENT LOCATION IN 3-D VELOCITY MODELS USING INITIAL AND SECONDARY ARRIVAL TIMES COMPUTED WITH DYNAMIC RAY TRACING**

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**ABSTRACT**

The objective of this project is to develop a seismic event location procedure that utilizes initial and secondary arrivals computed with dynamic ray tracing. We are developing software to compute travel times for complex three-dimensional velocity structures based on the dynamic ray tracing method that is used to compute Gaussian beam seismograms. Travel times for regional phases, such as Pg, Sg, Pn and Sn will be computed efficiently with the paraxial ray approximation, which requires ray tracing only to a point near the source or receiver. The derivatives of the travel-time field with respect to Cartesian coordinates can easily be determined from a single paraxial ray. We will use these derivatives in a standard least-squares iterative inversion for the event location. It will be determined whether the use of later phases improves on the location from first arrivals alone. We are designing this location code to be fast and easily automated. It will be available as a separate program and as an extension to the location program LocSAT.

**Key Words:** event location, 3-D models

**OBJECTIVE**

Reliable event location is an essential part of the Comprehensive Nuclear-Test-Ban Treaty (CTBT) monitoring effort. The objective of this project is to develop a seismic event location procedure that utilizes initial and secondary seismic arrival times computed with the Gaussian beam method. Software to compute travel times based on the dynamic ray tracing method that is used to compute Gaussian beam seismograms for complex three-dimensional velocity structures will be developed. This software will be integrated with existing hypocenter inversion software to produce new event location programs. The project will use the iterative least-squares inversion method.

**RESEARCH ACCOMPLISHED****Significance of the Problem**

High confidence event location plays a critical role in the identification and characterization of seismic sources. It is also an area of ongoing research, since the goal of routinely locating events to within an area of 1000 km<sup>2</sup>, the maximum practical area for timely onsite inspections, has not been achieved for all regions and event sizes. The location of low magnitude events remains a technological challenge. Low threshold events can only be recorded at regional distances. Thus their seismic waves spend relatively more time in the crust and upper mantle, where the lateral changes in seismic velocities can significantly affect travel times. In addition, smaller events are frequently recorded only at higher frequencies, where crustal reverberations and the interference of surface reflections make phase identification more difficult. Researchers are constantly improving velocity models by incorporating more complex structure, including laterally heterogeneous velocities and detailed Moho topography. In response, location programs have begun to take advantage of these new high-resolution three-dimensional models.

These location programs have to compute travel times between an event hypocenter and the recording stations for seismic phases traveling through the three-dimensional models. A common approach is the finite-difference method. By finite-differencing the eikonal equation of ray tracing, wave fronts and travel times can be extrapolated from point to point throughout a three-dimensional grid. After travel times from all possible source locations to all recording stations have been computed, the grid is searched for the event location that minimizes the difference between the computed and the observed travel times, using one of several norms. Only the first arrivals at the recording stations are used in the inversion for the hypocenter.

The program QUAKE3D [1] is an example of this type of a finite difference location program. It computes the first arrival only from all possible hypocenters, whether it is a geometric ray, refraction, or diffraction. The source location is found that minimizes the difference between the observed and computed first arrival times, using either the root-mean-square (L2) norm or the absolute value (L1) norm. The program 3DGRIDLOC [2] uses the same finite difference method to compute the travel times, but instead of the L1 or L2 norm, it uses the non-linear inversion algorithm of Tarantola and Valette [3] to find the best hypocenter.

Ray tracing is an alternative to the finite-difference method of computing travel times for three-dimensional velocity models for use in location programs. A technique called the shortest path ray tracing algorithm [4] has been used to compute travel times for a location program [5]. This method computes the first arrivals by tracing rays from all grid points to all recording stations. Then the inversion method of Tarantola and Valette is used to find the hypocenter.

A major drawback of these three programs is that no information about secondary arrivals is included in the location algorithm. The first arrivals calculated might not always be observed. Secondary arrivals that could help determine the event depth, an important parameter in discriminating between earthquakes and explosions, are not used.

A third method of computing travel times for three-dimensional velocity models is the reflectivity method. The reflectivity method computes complete seismograms with all secondary arrivals. Kennett [6] has recently used this method to constrain source depths and source mechanisms for small events. Although this technique shows

promise for determining source depth and mechanism, it is not meant to determine the complete hypocenter. The computations are also very time-consuming compared to finite-differencing or ray tracing.

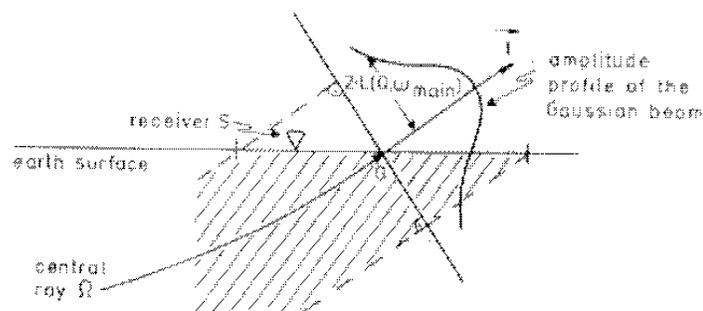
Clearly, a location method that uses initial and secondary arrival times for three-dimensional velocity models would complement the existing location programs. We are developing such a method by combining the paraxial approximation, which is used in Gaussian beam ray tracing techniques, with a standard hypocenter inversion method.

### **Accomplishments**

We are developing a seismic event location program that computes travel times for a three-dimensional velocity model using the Gaussian beam ray tracing method of Cervén [7]. These travel times are used in the nonlinear iterative least square inversion method of Jordan and Sverdrup [8] to compute the hypocenter. This is the inversion method used in the location program LocSAT, which is used by the Center for Monitoring Research. We will add an option to this program to use travel times and travel time derivatives computed by Gaussian beam ray tracing.

The Gaussian beam technique [9] can compute accurate seismograms for isotropic Earth models of varying complexity. It combines the ray concept with elements of wave theory. Instead of solving numerically the full wave equation or the eikonal equation as in finite differences or finite elements, a high frequency approximation is made to the equations of motion to obtain the parabolic wave equation whose solutions include Gaussian beams. The technique is at least an order of magnitude faster than the reflectivity method, the finite element technique and the finite difference method, especially when it is applied to models incorporating complicated lateral structures. Like WKBJ, it is possible to code in individual seismic phases separately, and therefore it is well adapted for use in inversion methods. It matches the widely distributed WKBJ code in speed without suffering the pitfalls at caustics that WKBJ encounters.

Since classical ray tracing fails at caustics, it would be very difficult to utilize it in a location procedure that

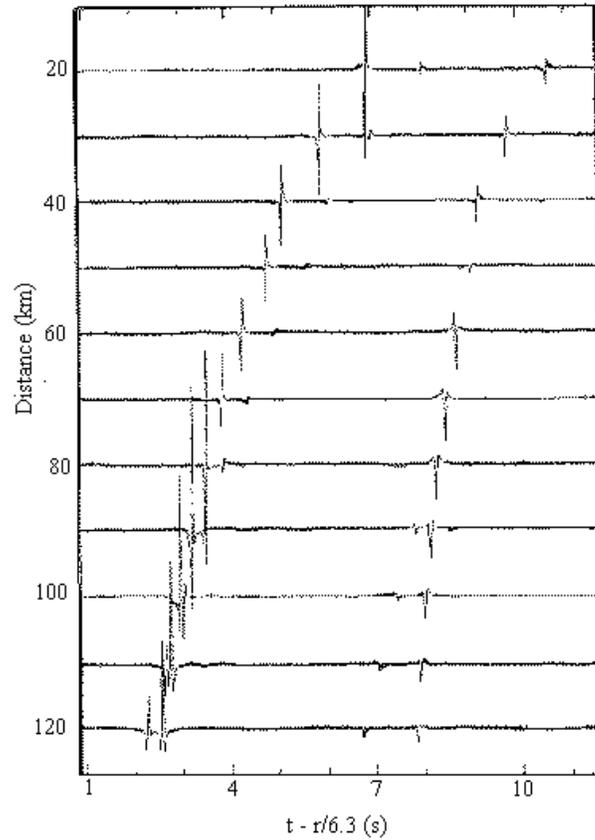


**Figure 1. The region around the central ray (shaded area) within which a receiver obtains a contribution from the beam. Adapted from Weber (1998).**

requires derivatives of travel time. The Gaussian beam method does not fail at caustics or triplications, since it forms phase arrivals from bundles of nearby rays instead of a single ray. This also means that there is no need for time-consuming iterations to find the exact ray from source to receiver. The beam is valid for any receiver that is within the beam width. (See Figure 1.) The travel times and the derivatives of travel time with respect to hypocenter coordinates are computed numerically from the Gaussian beam, instead of from a single ray, to insure that the location procedure is stable in the vicinity of caustics.

Another advantage the Gaussian beam method has over ray tracing is that it is capable of producing refractions or head waves. The technique can always compute the arrival time of head waves, such as Pn and Sn. In slowly varying media, the beams can be broad and the amplitude of head waves can also be computed. Figure 2 shows

an example seismic-section of Gaussian beam synthetics for a homogeneous three layer moho model. The head



**Figure 2. Gaussian beam synthetic seismograms for an explosive source in a two-layer crust plotted with a reducing velocity of 6.3 km/3. Phases include reflections off the crustal interfaces as well as converted phases, but not the direct P wave.**

waves become apparent at the larger distances. This amplitude information will be very useful for the seismic analyst.

As a major part of this project, we have developed codes to compute travel times and travel time derivatives based on the dynamic ray tracing method that is used to generate Gaussian beam synthetic seismograms for three-dimensional structures. This task is a straight-forward implementation of the three-dimensional method of Cerven\_ [7], starting with the two-dimension Gaussian beam program of Weber [10], as rewritten by Davis and Henson [11]. In the two-dimensional code, the velocities  $V_p$  and  $V_s$ , the density and the attenuation constants  $Q_p$  and  $Q_s$  are defined at each vertex of a grid of triangles. The analytical solution to the differential equations describing the behavior of the beam inside each triangle is used to trace the beam from source to receiver. In a similar manner, the three-dimensional model is a grid of tetradedra, and the velocities, density and attenuation constants are defined at each of the four vertices.

The travel time along a ray is computed by adding the contributions from each tetrahedron that the ray passes through. The equations for general three-dimensional dynamic ray tracing are given by Cerven\_ [7]. We use dynamic ray tracing to efficiently compute the ray and travel time between two points in the model. To

determine the ray from the source to the receiver, we start with an initial guess for the slowness vector at the source, trace the resulting ray and then use dynamic ray tracing to adjust the slowness vector. The convergence to the correct slowness vector is very rapid.

For the second technical objective of this project, we have extended the graphical interface that has been developed for the two-dimension Gaussian beam software [11] to display the input three-dimension models and display the ray paths that make up the beams. The OpenGL library, which is available on most computer platforms, including SGI, SUN and Windows95, is used to construct the 3-D displays. This work is essential to confirm the correct operation of the ray tracing and the gridding of the three-dimensional velocity structures.

The final objective of this project is to integrate the Gaussian beam software with the LocSAT location program. This program uses the iterative least-squares method of inverting for the hypocenter and therefore requires the partial derivatives of travel time with respect to hypocenter coordinates. Software for numerically computing these derivatives for the Gaussian beams will be developed along with routines to interface with the LocSAT code.

### **CONCLUSIONS AND RECOMMENDATIONS**

There are several results from this project. The first result is a program that computes travel times and travel time derivatives based on the dynamic ray tracing method that is used to compute body-wave Gaussian beam synthetic seismograms for three-dimensional velocity structures. The input to the program is a three-dimensional Cartesian grid of velocities, density and attenuation constants that describe the velocity structure. Also input is a list of seismic phases, specified either using standard terminology (pP, etc.) or by specifying the travel path (transmitted, reflected, refracted, converted, etc.) Finally, the source location, the source moment tensor and the receiver location is input. The output from the program is written in CSS3.0 format.

The second result from this project is a graphical interface which is a modified version of the program Xgbm [11], that displays the input 3-D velocity grid and the ray paths from source to receivers. The user is able to display ray paths for selected phases and also design custom ray paths.

The third result of this project will be a modified version of the location program LocSAT that is linked to a library containing the 3-D Gaussian beam travel time components. There will be an option to LocSAT to use the Gaussian beam travel times and travel time derivatives instead of the IASPEI tables. Both types of travel times will be available, i.e. the user will be able specify that individual phases use either the IASPEI tables or the Gaussian beam travel times for the hypocenter inversion. The Gaussian beam travel times and derivatives will be dynamically computed from the input 3-D grid at each iteration of the hypocenter location.

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