JOINT INVERSION OF RECEIVER FUNCTIONS AND SURFACE-WAVE DISPERSION FOR CRUSTAL STRUCTURE

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ABSTRACT

A program for joint inversion of teleseismic receiver functions and surface-wave dispersion measurements has been implemented by merging existing inversion programs. The purpose of this effort is the implementation of an algorithm that draws upon the true capabilities of each technique in a manner that complements the imperfections of the other. Inversion of single-mode surface-wave dispersion yields a velocity model that cannot resolve crustal velocity discontinuities; the receiver function technique excels at defining impedance changes and travel times but suffers from tradeoffs between velocity and depth. The combination of the two yields a velocity model that is better constrained.

The inversion program uses an a priori participation factor p such that the end members p = 0 or 1 represents an inversion using only one set of observations (receiver function or surface-wave dispersion). We present a formulation that compensates for the different units of the two classes of observations and the number of observations to yield formal, statistical estimates of model error.

The program is tested using surface-wave group velocity dispersion estimated from a surface-wave tomography study in the Arabian plate and teleseismic receiver functions computed usijng seismograms from permanent and portable broadband stations in the region. Initial results are encouraging. Experiments with synthetic seismograms show great potential for resolving relatively fine-scale vertical structure of the crust and crust-mantle transition.

The tool developed here will be available for use in constraining regional structure at existing and planned broadband stations deployed as part seismic monitoring under the CTBT.

Key Words: Surface wave, receiver function, crustal structure, algorithms

OBJECTIVE

The object of this effort is to develop a stable inversion program that simultaneously extracts constraints on regional crustal structure from both surface-wave dispersion curves and teleseismic receiver functions. This work is similar to that performed by Özalaybey et al (1997) but the bandwidth of surface-wave observations is greater and the surface-wave dispersion estimates are extracted from earlier tomographic imaging results. Separate inversions of the two types of data lead to models that contain artifacts that reflect subjective inversion constraints, such as a minimum length or roughness, rather than actual earth structure.

The research community has almost 40 years of experience with computer assisted inversion of surface-wave dispersion data. We have learned that inversion of fundamental-mode dispersion over a limited frequency band is an underdetermined problem. Models consisting of a few constant velocity layers or models consisting of many thin layers can be found that fit the same observations. One would hope that there is similarity in some gross features of these models. It is our experience that surface-waves can constrain mean crustal velocity to a given depth, but cannot easily resolve velocity discontinuities or gradients.

Teleseismic P-wave receiver function inversion has become a standard tool for crustal structure estimation. The receiver function is most sensitive to impedance contrasts, which give rise to converted phases, and to phase travel times, but allows for substantial tradeoff between the depth and velocity above an impedance change.

Joint inversion of these complementary observations should reap the benefits of each, helping to reduce the imperfections of either. The receiver function will tell us something about the nature of strong changes in impedance, such as at the crust-mantle boundary, and the crustal travel times while the surface-wave crustal velocity estimates will permit mapping these times into depth.

The reason this approach is of value to the CTBT program is that the crustal structure beneath seismograph stations can be defined. Work by Ritzwoller et al (University of Colorado) under previous funding has given the research community regionalized surface-wave dispersion curves for much of Eurasia. As CTBT monitoring stations are installed, teleseismic waveforms can be collected during the initial operation (depending on frequency of worldwide large earthquakes) to provide the receiver functions for an estimate of the crustal structure beneath the station.

In the long run, improved estimates in the crustal structure arising from the joint inversion of receiver function and surface-wave dispersion will help improve event location accuracy. For small event analysis, the incorporation of receiver function observations with longer period (> 15 seconds) surface-wave observations typical of tomographic studies could help produce more accurate filters for signal identification and enhancement and eventual source identification. We also believe that this localized structure will permit waveform inversion of regional signals for source identification. The need for a good crustal structure is required for both.

RESEARCH ACCOMPLISHED

1. Design of Goodness of Fit Criteria

The design of an inversion program relies on the awareness that the data,, dispersion and receiver function, have different units, magnitudes, noise and number of observations. Yet we desire in some sense to give each data set equal influence on the final model.o balance the relative importance of each set of observations, we use a parameter p to control the participation of each data set. For a value of $p=0$, only the receiver function data are used, while a value of $p=1$ will mean that only the dispersion data are used..

Specifically the following goodness-of-fit functional is minimized iteratively:

$$S(\vec{x}) = \left[(1-p)N + pM\right]\left[\frac{(1-p)}{N}\sum_{i=1}^{N}\left(\frac{y_i - Ax_i}{\sigma_{y_i}}\right)^2 + \frac{p}{M}\sum_{j=1}^{M}\left(\frac{z_j - Bx_j}{\sigma_{z_j}}\right)^2\right]$$

In this expression the model perturbation for an iteration is given by **x,** N receiver function and M dispersion residuals are given by **y** and **z** respectively. The A and B matrices contain the partial derivativesof the receiver function and the surface-wave dispersion observations with respect to shear-wave velocity. The σ values are estimates of the data standard errors of each data set. This formulation does a number of things: 1) the normalization by the standard error, accounts for the different units and magnitudes of each data set; 2) the division of the normalized residuals by N and M before the summations corrects for the different number of observations within each data set; 3) the coefficent *[(1-p)N+pM]* forces the expression to equal that expected for a stochastic inverse for the end member cases of $p=0$ and $p=1$. This requirement means that the model variance – covariances are obtained directly from the inverse matrix of the normal equations. The formulation can also be viewed as a weighted least-squares procedure. The σ's can be estimated from residuals of fit at each iteration with a minimum positive value for safety and realism. Note that by construction, $p=0.5$ means that each data set has equal influence on the final model.

The present formulation can be easily extended. For example, we can use a damped inversion scheme for stability. We can also apply a smoothing criteria for the model perturbations. Finally the functional could be generalized more by permitting inversion of other data, such as refraction times or event waveforms.

2. Design of Inversion Program

The joint inversion procedure builds upon two existing programs: *SURF* (Herrmann, 1987) for surface-wave dispersion inversion and *BODY* (Ammon, 1990) for receiver function inversion. Julià (1999) developed the program *JOINT* to connect these procedures. The use of the programs and data flow are given in Figures 2.1-2.3.

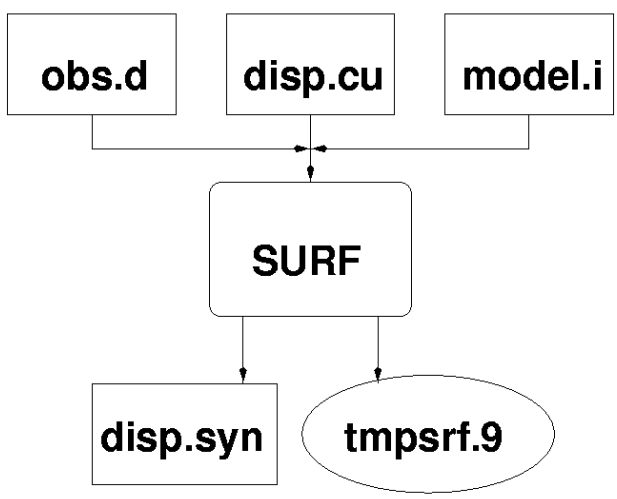


Fig. 2.1. Data flow for the program *SURF*. tmpsrf.9 contains the partial derivatives.

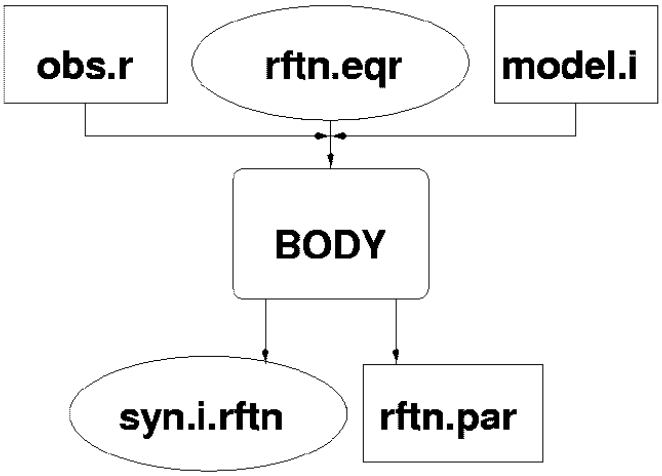


Fig. 2.2. Data flow for *BODY*. Rftn.par contains the partial derivatives

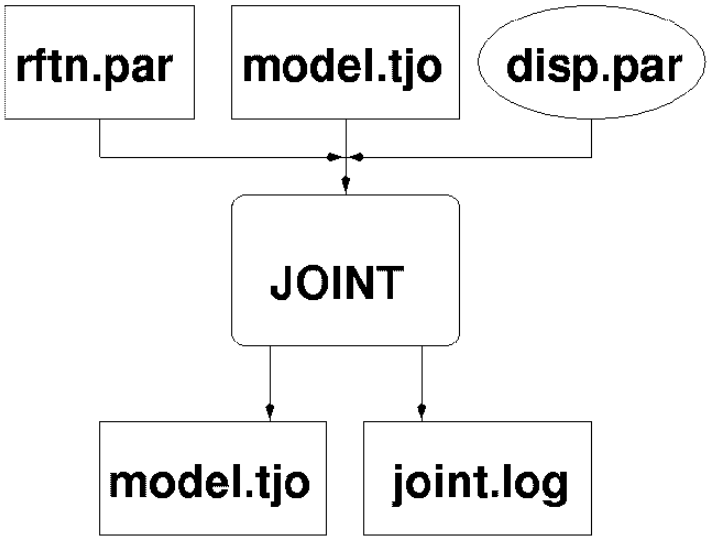


Fig.2.3. Data flow for the program *JOINT*. Model for current iteration is *model.tjo*.

3. Application to Arabian Plate

Seismic wave propagation in the Arabian Plate has been the topic of a number of investigations (Mooney et al, 1978; Sandvol et al, 1999; Mokhtar et al, 1999). To demonstrate the inversion technique we use the surface-wave dispersion observationsof Mohktar et al (1999) and the receiver functions for the station SODA which was part of a special IRIS PASSCAL deployment between November, 1996 and February 1997. Figure 3.1 shows the Arabian plate and the location of the PASSCAL deployment.

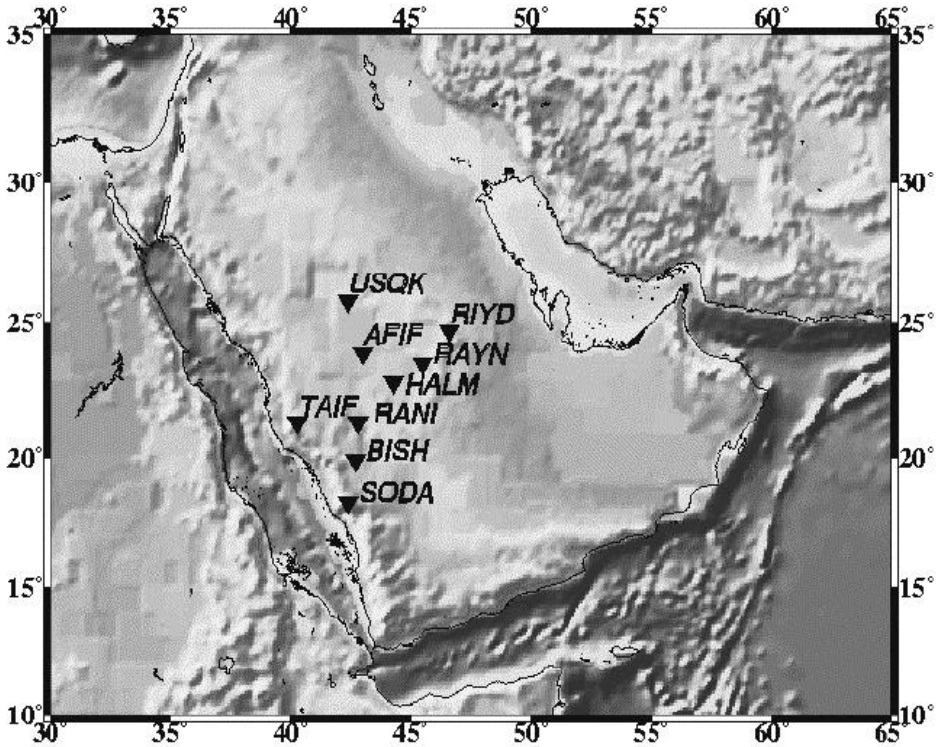


Fig 3.1 Arabian Plate and IRIS station locations.

The station SODA is used for receiver functions since it lies close to the Saudi Arabia refraction line for which crustal models have been published (Mooney et al, 1978). Four teleseisms with magnitudes between 6.1 and 7.0 were used to compute the receiver functions using the methodology of Ammon (1991) and Ammon et al (1990). Mokhtar et al (1999) studied Rayleigh – and Love-wave group velocity dispersion within the Arabian Plate by performing a tomographic inversion using small cells and a spatial smoothing constraint. We use the Rayleigh-wave group velocity dispersion for the cell that contains the station SODA.

Separate inversion were performed using values of *p=0.0 (receiver function only and 1.0 (surface-wave dispersion only).* As expected, the fits to each individual observation set were acceptable, but the complementary observations were not matched by the two extremes of the participation factor. In Figures 3.2 to 3.4 we show the inversion results for values of *p=0.1, p=0.6,* and *p=0.9*., respectively.

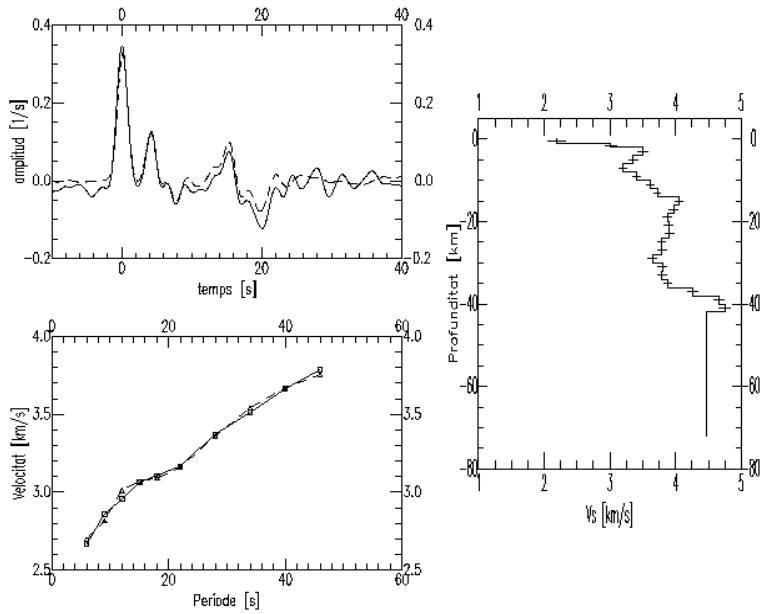


Fig. 3.2. Simultaneous inversion results for *p=0.1.* The receiver function is at the upper left, the surface wave dispersion at the lower right, and the model at the right. The solid line denotes the data and the dashed line the predictions for the model at the right. The fit to both sets of observations is not too bad, but the model has several fluctuations and a rather high mantle velocity

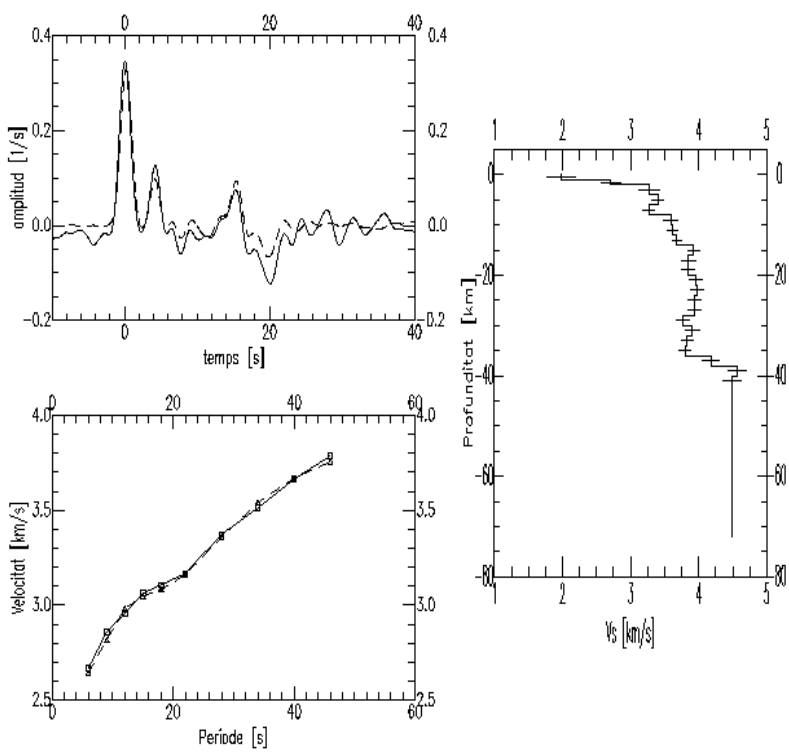
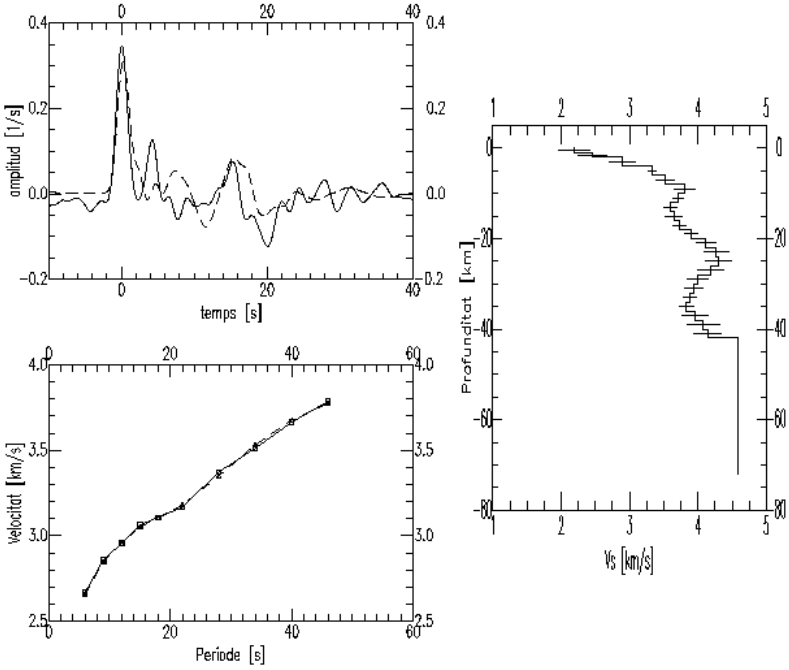


Fig. 3.3. Processing results of *p=0.6* which corresponds to a slightly more favorable weight for the surface-



wave dispersion measurements.

Fig. 3.4. Inversion results for *p=0.9.* For this value, the surface waves are dominant as evidenced by the excellent match to these observations, but a poor match to the receiver function.

In Figure 3.5 we compare the joint inversion results for *p=0.6* with the structure estimated using a refraction profile (Mooney et al, 1978). The results of the joint inversion are very consistent with those obtained in the earlier refraction analysis. Although there are some small fluctuations in the velocity of the thin layers, the overall comparison is quite good. The crustal thickness is particularly consistent, about 40-42 km. Interesting differences are also noticeable. The model from this study has a well-defined, thin near-surface layer with lower velocities and the transition from the crust to mantle appears relatively sharp (which is consistent with the strong P-S conversion in this region seen in the receiver function.

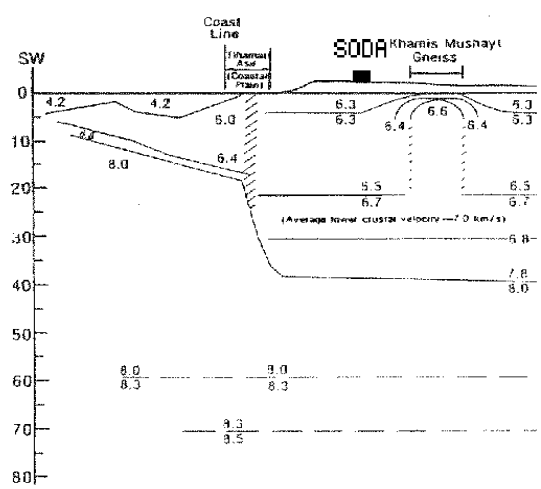
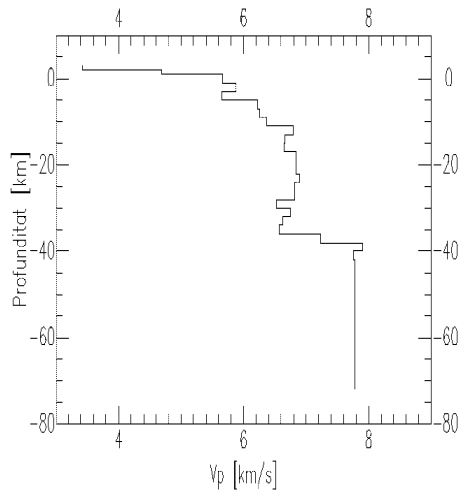


Fig. 3.5 Comparison of *p=0.6* inversion to Mooney et al (1978) model.



CONCLUSIONS AND RECOMMENDATIONS

This effort has shown that the joint inversion procedure works as expected. Our method differs from that of Özalaybey et al (1997) primarily in the bandwidth of the surface waves and in a goodness of fit criteria, which expilicitly accounts for the different units of the two iobservations (in this case *km/sec* and sec^{-1} for the dispersion and receiver function, respectively). We also include an *a priori* parameter thatcontrols which data set has the most influence. In this exercise, many thin layers were used without a differential smoothing constraint. That the *p=0.6* choice leads to a model that fits the data without perhaps artificial low velocity zones indicates the robustness of the technique.

The success so far supports our initial hypothesis that the joint inversion will produce a more tightly constrained earth model than those arising from separate inversions of either of the two data sets.

During the second year of this effort, Dr. Jordi Julià will join us as a post-doc researcher under another project. He will be available to document the programs and further test the technique with real data sets. The resulting program will be available from the authors as part of their software documentation efforts.

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